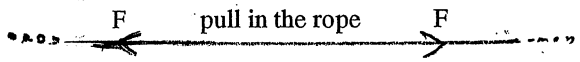
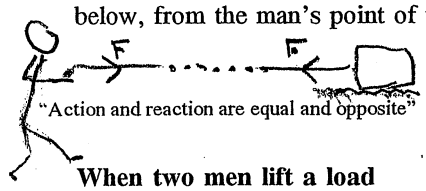
Pulleys

ROPES PULL

The tension (tightness) in a rope pulls both ways with equal force F:

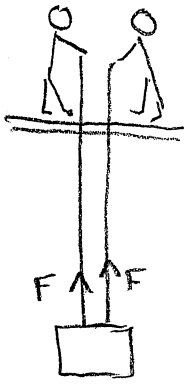


Arrows show force: above, from the rope's point of view; below, from the man's point of view, and the load's.



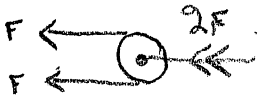
When two men lift a load with separate ropes each with force F they make a double force, 2F.

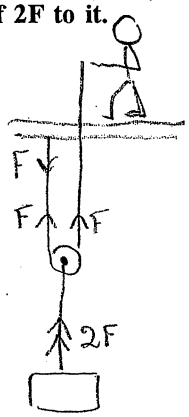
If each man pulls up a metre of rope, the load lifts by one metre.



PULLEYS

A pulley is a machine reversing the direction of F at the pulley, so that the rope pulls on the pulley twice and applies a force of 2F to it.

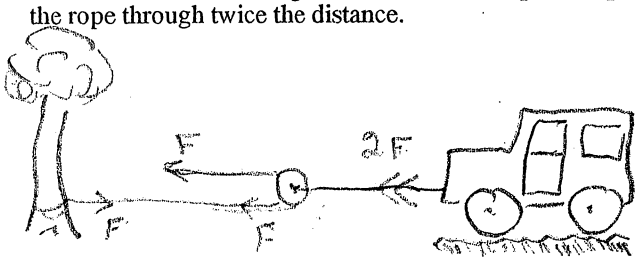




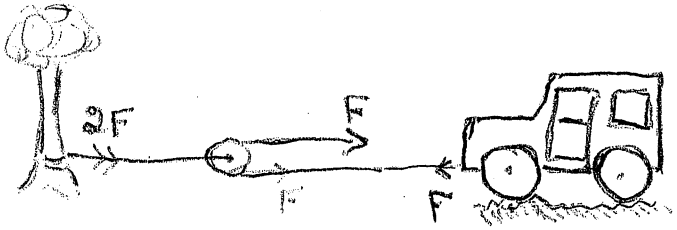
The man pulls the rope through double the distance by which he lifts the load, so he uses twice the length of rope. "We don't get something for nothing..."

PULLING A BOGGED CAR

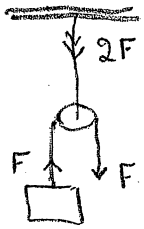
If one end of a rope through the pulley is anchored to a tree, the tree resists, with force F, against the pull F at the other end of the rope, and the result is a pull of 2F on the pulley, hence on the car. The doubling of the force is at the price of pulling the rope through twice the distance.

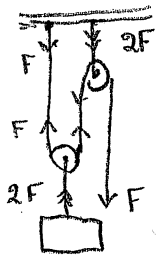


If the pulley is tied to the tree and not to the car, there is double the pull 2F on the tree, but only F on the car, and the rope is pulled the same distance as the car.



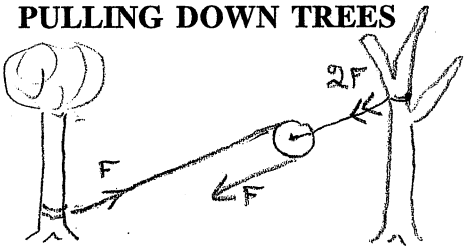
LIFTING A WEIGHT



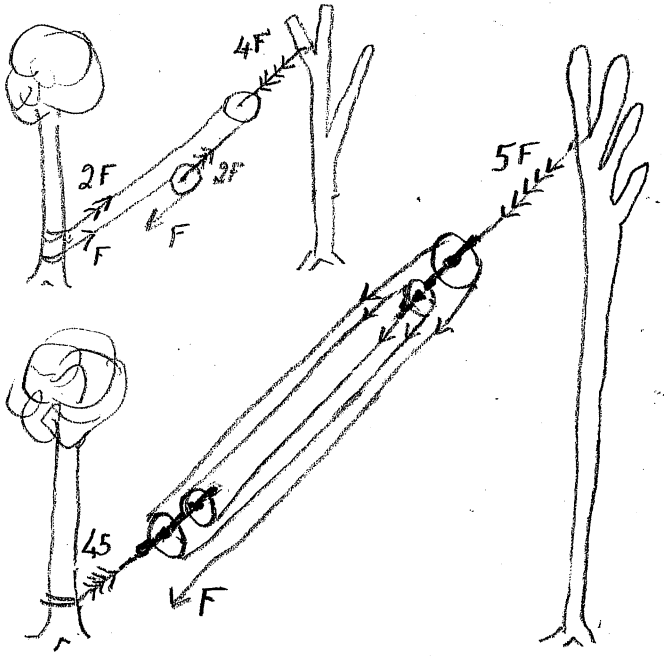


The pulley tied to the ceiling does not move with the load. It multiplies the force on the ceiling, not on lifting the load. However, it can be advantageous to pull from below.

PULLING DOWN TREES



An anchor tree is strongest at the bottom. A dead tree gives the best leverage near the top. But caution! Anchor trees have been known to pull up by the roots, and dead tree trunks can break off at the top.



Pulley wheels mounted side by side (or with a big wheel and a smaller one underneath) are called a **pulley block.** A rope and two pulley blocks is called a **block & tackle.**

The block & tackle with the double pulley blocks multiplies the force by 5 by using 5 times the length of rope.

When the two pulley blocks come together, they are "chock-a-block", and have to be repositioned, which can be dangerous if the dead tree is leaning the wrong way.

MECHANICAL ADVANTAGE

The number of times the force is multiplied is called the mechanical advantage. It is the same as the number of ropes which pass through the moving pulley.

The total force on the anchor is the number of ropes through the fixed pulley block. This number is one less than on the moving pulley block.

A multiplication of force by 5 means that the original force must pull through 5 times the distance.

Square Roots

THE SQUARE of a number is that number multiplied by itself. It is the area of a square figure.

For example, $3 \times 3 = 9$ and it is written as $3^2 = 9$
and $4 \times 4 = 16$ and is written as $4^2 = 16$.

Taking **the square root** of 9 means finding what multiplied by itself becomes 9. We write it as $\sqrt{9} = 3$ and $\sqrt{16} = 4$.

They are the easy ones! What about the square root of numbers like 2, 3 and 5 ? Can they be calculated without an electronic calculator (or logarithm tables)? Yes, square roots can be calculated by a **long division method** — which revises the multiplication tables and careful setting-out, so neglected in the modern over-dependence on calculators and electronic gadgetry.

An example: find the square root of 7,829,216

The setting out is akin to a **long division.** However, separate the digits into pairs: 7 82 92 16. Work from the ***right hand end,*** from the decimal point. If there are figures after the decimal point, pair them off also from the decimal point.

The reason? Squares of single digit numbers 4,5,6,7,8,9 have two digits from 16 to 81, whereas the squares of single digit numbers 0,1,2, 3 are themselves only one digit numbers but written as two digits, with 0 in front, hence 00, 01, 04, 09.

The actual calculation of $\sqrt{7,829,216}$

Now work from the ***left hand end.*** The nearest perfect square to 7 (and below it) is 2, since $2^2 = 4$, The first digit of the answer is **2** (at right, bold), and remainder 3, set out thus:

2) 7 82 92 16 (**2**

$2 \times 2 = 4$. So subtract 4: -4

3

Bring down the next pair, so the 3 becomes 382; double the 2 of the first figure of the answer, and write 4 as part of the new divisor (see below). Now supply an extra digit on the right of the 4, also supply the same digit to the right of the preliminary answer. Try 7. Multiply by that 7 (notice it is being squared). For beginners, this step will be trial-&-error, and a trying out of several of the digits between 0 & 9:

Double 2→4 and try supplying 7: 47) 382 (**27**

$47 \times 7 = 329$. Subtract it: -329

Bring down the 92. 5392

Double 27→54. Supply a 9: 549) 5392 (**279**

Subtract 9×479: -4941

and bring down 16: 45116

Double 279→558. Try × 8: 5588) 45116 (**2798**

-44704

412

In 7,829,216, no digits are after decimal point, so supply 00 and bring it down. Double 2798→5596. The only multiplier that produces something less than 41200 is 0:

55960) 41200 (**2798·0**

-0 and bring down 00.

Double 27980→55960. Try 7:

559607:) 4120000 (**2798·07**

Subtract it: -3917249

Bring down another 00: 20275100

Double 2798.07→559614 and try × 3:

5596143) 20275100(**2798·073**

Subtract it: -16788429

Call it off with a remainder: 3486671...

Answer: $\sqrt{7,829,216}$ = **2798·073**

Check on a calculator: 2798·0736 (one more figure).

Second example, without rubrics: $\sqrt{5}$

2) 5 (2

4

42) 1·00 (**2·2**

84

443) 1600 (**2·23**

1329

4466) 27100 (**2·236**

26796

44720) 30400 (**2·2360**

447206) 3040000 (**2·23606**

2683236

Remainder 356764

Answer: $\sqrt{5}$ = **2·23606**

The calculator gives two more digits: 2·2360679.

ALGORITHM — how and why it works...

$(a + b + c + d \ldots)^2$

$= a(a) + b(2a + b) + c[2(a + b) + c] + d[2(a + b + c) + d] \ldots$

where pro-numerals a, b, c, d etc are progressively extracted as the coefficients of the descending powers of 10 (from millions to hundreds etc) to give the answer. Note the progressive doubling of more and more digits.

Pope Benedict XVI on God and mathematics

at an interview with young people in 2005

THE POPE told his young questioners that: "The great Galileo said that **God wrote the book of nature in the form of the language of mathematics.** He was convinced that God has given us two books: the book of Sacred Scripture and the book of nature. And the language of nature — it was his conviction — is mathematics, so it is a language of God, a language of the Creator.

"The surprising thing is that this invention of our human intellect [i.e. mathematics] **is truly the key to understanding nature, that nature is truly structured in a mathematical way, and that our mathematics, invented by our human mind, is truly the instrument for working with nature, to put it at our service, to use it through technology.**

"It seems to me almost incredible that an invention of the human mind and the structure of the universe coincide. Mathematics, which we invented, really gives us access to the nature of the universe and makes it possible for us to use it.

"Therefore, the intellectual structure of the human subject and the objective structure of reality coincide: the subjective reason and the objective reason of nature are identical. I think that this coincidence between what we thought up and how nature is fulfilled and behaves is a great enigma and a great challenge, for we see that, in the end, **it is 'one' reason that links them both.**

"One reason could not discover this other reason were there not an identical antecedent reason for both. In this sense it really seems to me that **mathematics** — in which as such God cannot appear — **shows us the intelligent structure of the universe.** Now there are also theories of chaos, but they are limited because if chaos had the upper hand, all technology would become impossible. Only because our mathematics is reliable, it technology reliable"...