***Cardinal Newman Catechist Consultants* — 18th October, 2015 — *HANDOUTS* n. 127**
"Clear, brief and easily assimilated by all"
This began as *Handouts* n. 25 over a hundred issues ago, way back in May, 2005, and is now finally completed

Maths for Muddlers

**for the less math's minded home-tutor/schooler/educator —
to encourage tutors and engender confidence for High School mathematics**

BE OF GOOD HEART!

THERE are still teachers telling pupils, "You don't have to understand it, you only have to know it."

This is half-true. It is also half-false.

- Half-true: knowing the multiplication table perfectly is far more important than an abstract understanding of it. And later, we can build on the parrot-fashion.
- Half-false: even barbaric and anti-intellectual! for God created us in His image, gave us intellect and understanding and our ability to reason.

Reflecting on my experience long ago, when Year 7 was called 1st year high school (later 'first form'), and most left school at what is now Year 9 and "we few" did a Leaving Certificate in Year 11, I can now see I was not 'ready' for *theorems* in geometry: they were too abstract, too theoretical, for me to understand.

I also see the lost opportunities: many things could have been taught more clearly; more links made with maths in daily life; more interest given to future usefulness; and cross-reference to other subjects.

MURMURERS NOT ALLOWED, at least NOT ALOUD

Tutors must make a **pre-emptive strike** against defeatism! They must **inoculate** pupils against the calumnies that maths is boring and useless.

The truth is that maths is good for you:

- It forms the mind for **hard thinking** and logic;
- is **intellectually challenging** for character.

Yet some are less suited for this type of academic mental work, and this must be respected.

So wherever possible, tutors should make maths:

- **understandable**
- **memorable**
- **interesting**
- **relevant**
- and give **special help** to those in despair!

If it's in reach of a pupil's understanding it should be understood. The academic should prove things (and prove that they can!), and be more certain to remember accurately and better able to apply it.

EXPERIMENT OR THEORY OR BOTH?

With the wisdom of hindsight, more help could have been given to 'the average student' who finds maths hard, indeed burdensome. **What such students need is something more hands on, i.e. experimental, and not so abstract or theoretical or philosophical.**

Scientific Method is **experimental:** observation and measurement, then intelligent guesses (hypotheses) to be tested by further experiments. Try it on maths. Start arithmetic, algebra and geometry with wood or cardboard **models; measure** with rulers & protractors.

This is an engineer's approach — pragmatic, and even callous with mathematical rules "as long as it works". Of course, this can degenerate into knowing without understanding, to using without proving. Certainly for the more academic, the most practical course pursues the subject for its own sake. But there's room for two approaches, where theory and practice support each other.

Pure maths proves things logically. It feels no need for confirmation by experiment. However, the average student finds this rigour too hard.

HANDS-ON ACTIVE PARTICIPATION

There should be lots of work with compass and straight edges, as well as set squares, protractors and rulers. Facility with these **tools** makes many minds more *facile* than the abstractions drawn on paper by someone else. Before attempting the theory in geometry, let alone trigonometry, the pupils should be able to bisect a line; to bisect an angle; draw a perpendicular to a line at a point on the line and also from a point not on the line, etc.

Whenever possible, pupils should make their own ***learning aids.*** Learn equations by making a beam balance; learn sines of angles by rotating a disk with a plumb bob on the edge; sine wave generators using a Meccano set or programmed into a computer.

See ***The Five Regular Polyhedra*** for the templates for making them, and profound ideas in a few words, in ***Bush Boys and Bush Rangers*** p. 55; and ***Pulleys, Levers and Cog Wheels, ibid.*** p. 209.

PUPILS CHECK THEIR WORK

No addition of a column of figures is acceptable unless it has been **done twice,** down the columns and then up the columns. No subtraction should be accepted until it has been checked by addition.

A single simple measurement is not acceptable on its own. In the real world of woodwork and plumbing etc, **"Measure twice, cut once"** is the motto.

A pupil must test an answer to an algebra problem by testing it with numbers: "Does it work for $x = 1$ and $y = 2$, etc. Overcome laziness with firm discipline: human life can depend on getting sums right: it's a life-or-death business! Discipline enforced in class helps self-discipline and a rarer virtue, *intellectual discipline,* by which the human will makes the mind go to work, whether for utility, money, curiosity or the greater glory of God.

GOOD HABITS for TUTORS and PUPILS

- **REVISE,** revise, revise the basics, **OFTEN,** often, often!
- Work must be neatly written and laid out;
- practise mental arithmetic: it's practical for living!
- put numbers in clear columns of units, 10s, 100s etc;
- show all working, abbreviate "because"/"therefore";
- $-$ $\times$ $\div$ calculations must be done twice as a check:-
- check + down a column, then up; of right, then left;
- check $-$ by adding *subtrahend* to the answer;
- check $\div$ by one digit by multiplying; otherwise do it twice;
- use superscript numbers for "scaffolding";
- find maths words in a dictionary (& Latin/Greek roots);

- dictionary derivations help remember abstract ideas:-
- plane figures: triangle, circle, rectangle, quadrilateral, parallelogram;
- solids: cube, sphere, tetrahedron, octahedron, dodecahedron, icosahedron;
- (p. 55 *Bush Boys and Bush Rangers*; number of sides + 2 = angles+ faces
- triangle: right-angled, isosceles, equilateral, scalene etc;
- co-ordinate geometry: abscissa and ordinates;
- calculus: differentiation and integration;
- trigonometry: sine, cosine, tangent, cosecant, secant, cotangent;
- (never violate the basic rule, "***one*** new idea at a time");
- avoid unnecessary mnemonics (see *Handouts* n. 19);
- rough definitions are OK at first:-
- e.g. ellipse/rhombus: circle/square pushed out of shape,
- parabola a sort of dangling chain (but not exactly).
- But witticisms like "snakes multiply by logs" are dated.

INSIGHTS for TEACHERS

MATHS STARTS with arithmetic. Arithmetic is a Greek word meaning 'number' though it is not a word much used in schools these days.

Arithmetic is the most useful part of mathematics, the four operation, + - × ÷.

Addition leads to multiplication, subtraction to division and also to a new dimension of thought, negative numbers.

Warning on what follows: it's not a beginners lesson, nor even a series of lessons. It is an **overview** for teachers, looking back over country already travelled.

ADDITION + is basic. Arithmetic begins with it and is called 'doing sums': e.g. $1 + 2 + 3 + 4 = 10$. The ***sum*** of several numbers means adding them up.

MULTIPLICATION is continuous adding on of the same number: $2 + 2 + 2 + 2 + 2 = \mathbf{5 \times 2 = 10}$.

EXPONENTS are the continuous multiplication of the same number: $2 \times 2 \times 2 \times 2 \times 2 = \mathbf{2^5} = 32$. In this example, the exponent (=power=index) of the number 2 is 5; i.e. 2 writ 5 times multiplied together.

INVERSE OPERATIONS

Addition, multiplication and exponents have **inverse functions,** that is, reverse operations that 'sort of go in the opposite direction'.

SUBTRACTION is the inverse function of addition: $5 - 2 = 3$. This leads to a great jump forward, with $2 - 5 = -3$ which starts us on **negative numbers.**

DIVISION is the inverse function of multiplication: $10 \div 5 = 2$. Thence another jump forward, a **fraction** $\frac{5}{10}$, i.e. five out of ten parts.

This fraction can be written as ½ or $\frac{1}{2}$; or a **percentage,** $50\% = \frac{50}{100}$ (% means 1/100); or it can be written as a special sort of fraction called a **decimal,** e.g. 0·5, which means $\frac{5}{10}$.

Decimals are fractions of 10 or 100 or 1000 etc, called tenths, hundredths, thousandths etc to as many 'decimal places' as are needed:
$0{\cdot}1 = \frac{1}{10}$ and $0{\cdot}01 = \frac{1}{100}$ and $0{\cdot}001 = \frac{1}{1000}$.
So the decimal of $\frac{1}{8} = 0{\cdot}125 = \frac{1}{10} + \frac{2}{100} + \frac{5}{1000}$.

LOGARITHMS are the inverse function of exponents. The number 10 raised to the exponent 3 is $10^3 = 1000$ and log 1000 to base 10 is 3: $\log_{10} 1000 = 3$. To multiply numbers, add their logs and look up antilog.

INTERESTING! SEE, TOUCH, THINK

I nearly added, 'tasting', because division and fractions are of course (and quite rightly) taught in terms of cutting up cakes for hungry children (Pavlov's Dog syndrome). Have a real cake and carve it up — for pupils who really try and don't whinge.

Straight **lines** make triangles; even doodling makes circles and sinusoids, and herewith begins geometry.

For practical examples/applications in maths use **physics:** hence basics of Length, Mass and Time; and units based on them, such as weight, work, power, volts, amps, ohms, hertz, i.e. integrate the curricula.

Even **grammar** comes into the simplest arithmetic: two plus three ***is*** five, two and three ***are five,*** because 'plus' is a preposition whereas 'and' is a conjunction, (hence singular subject, singular verb, etc).

Congruent triangles can be taught in terms of not leaning your chair back on two legs! Ask me how.

Bicycles: count the teeth on each cog, (it is not easy to mark the tooth on which you start counting). Tabulate them. Divide the number on front cog by number on back cog. Claims to have 21 gears are misleading if there are actually only 9 *different* ratios.

Hobby-Farms: areas, dimensions, contours; rain gauge, run-offs from roofs, how much rain to fill tanks.

Outdoor sports: finding north; measure shadow sticks; solar system; sun and seasons; time by sun.

Artistry: geometric patterns with compass and ruler, right through to chaos theory (and the delights of $i = \sqrt{-1}$, unreal and compound numbers).

Car engines: Consider the 'dwell angles' on fly wheels; gear ratios from manufacturer's manual; calculate maximum slope up which an engine's torque could drive the car. Torque means twisting force. Make graphs of torque against revolutions per minute (rpm). Internal combustion engine needs clutch and gear box, but steam engines and electric motors do not. Why?

Aeroplanes: see Newsletter 173/7 of 1st May, 2002, for the 'one in sixty rule' as a rough radian measure., which assumes $\pi = 3$ approximately. More accurate navigation needs trig for 'spherical triangles' in which sides are themselves in angular measure.

Make the idea of π (pi) more real by measuring diameter and circumference of coins, jars and other cylinders. There are also some practical problems to be solved in measuring around a curve. Use string, or a strip of paper, or roll things along paper. Then divide the circumference by the diameter. Discover the approximation 22/7.

Pythagoras' Theorem: discover it by measuring right angled triangles: easy ones at first, with sides proportional to 3-4-5; next, general cases. Then jigsaw puzzles; with pupils actually making them (see *Handouts* n. 18). Finally, do a geometric proof, which depends on a number of earlier theorems.

Finally, only allow **calculators** when pupils have proven they can do sums accurately without them Then they are they entitled to power tools.

Dear Reader, you are welcome to ring me for help: Father James Tierney: phone/fax 02 4829 0297