

Rhythms in Reciprocals and Verses

Marvels from God in the Mind of Man

RHYTHMS in RECIPROCALs

A RECIPROCAL is "one over a number." When two boys share a cake, they should get half a cake each. If an extra boy comes, and each gives him a sixth, they get a third each. These fractions can be expressed as decimals, $\frac{1}{6}, \frac{1}{2}, \frac{1}{3}$.

A decimal is a sort of fraction based on tenths. The decimal fraction of a half is easy. It is five tenths, $\frac{5}{10}$, and written 0.5. Some fractions have tenths and hundredths and thousandths, and so on. Decimals make the four basic operations, + - × ÷, easier.

A third, $\frac{1}{3}$, is a tricky decimal that goes on forever: $\frac{1}{3} = \frac{3}{10} + \frac{3}{100} + \frac{3}{1000} \dots = 0.333333\dots$ in shorthand, $0.\dot{3}$, where the 3 has a dot on top of it, and is pronounced "nought point three repeater".

Reciprocals of prime numbers (and of the multiples of prime numbers) are repeating decimals with all or some digits repeating: thus the reciprocals of 9 and 11: $\frac{1}{9} = 0.1111\dots = 0.\dot{1}$ and $\frac{1}{11} = 0.090909\dots = 0.\dot{0}9$.

The decimal of $\frac{1}{6}$ stranger since its first digit does not repeat: $\frac{1}{6} = \frac{1}{10} + \frac{6}{100} + \frac{6}{1000} \dots = 0.16666\dots = 0.1\dot{6}$.

To turn $0.\dot{3}$ back into a fraction, let $x = 0.\dot{3}$.
 $\therefore 10x - x = 3.3333\dots - 0.3333\dots = 3$,
 so $9x = 3$. $\therefore x = \frac{1}{3}$.

Fractionalize $0.1\dot{6} = 0.16666\dots$. Let it be x . So $10x = 1.6666\dots$ and $10x - x = 1.5$ since the decimal tails 6666... subtract off.

$\therefore 9x = 1.5$ so $90x = 15$, $\therefore 0.1\dot{6} = \frac{1}{6}$.

The reciprocal of 7 is $\frac{1}{7} = 0.142857142857142857\dots = 0.\dot{1}42857$ (all digits from dot to dot repeat).

It is trickier to turn $0.\dot{1}42857$ back into a fraction: let $x = 0.\dot{1}42857$. $\therefore 1,000,000x = 142,857.142857$
 $\therefore 1,000,000x - x = 142,857$ (the decimals subtract off)
 $\therefore 999,999x = 142,857$. Now $\frac{99}{14}$ is roughly 7 so we try it as a factor: $7 \times 142,857$ equals 999,999 exactly;

\therefore the fraction required is $\frac{1}{7}$.

Reciprocal of 7
$\frac{1}{7} = 0.\dot{1}42857$
$\frac{2}{7} = 0.\dot{2}85714$
$\frac{3}{7} = 0.\dot{4}28571$
$\frac{4}{7} = 0.\dot{5}71428$
$\frac{5}{7} = 0.\dot{7}14285$
$\frac{6}{7} = 0.\dot{8}57142$

- The same six digits 1 4 2 8 5 7 occur in each of the repeating decimals.
- $14 \times 2 = 28$ and $28 \times 2 = 56$ and $56 + 1 = 57$.
- As expected, the first decimal digits of each fraction are in ascending order, 1,2,4,5,7,8.
- Each has the same cyclic order of $\frac{1}{7}$ but with a new starting point, thus $\rightarrow \rightarrow$
- There is a symmetrical interchange of the first three digits with the last three, between these pairs $\frac{1}{7}$ and $\frac{6}{7}$, $\frac{2}{7}$ and $\frac{5}{7}$, $\frac{3}{7}$ and $\frac{4}{7}$.
- Each pair add up to $0.99999\dots = 0.\dot{9} = 1$.



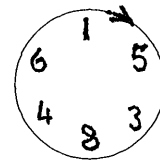
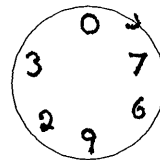
Next, the reciprocals of 13 and their multiples:

$\frac{1}{13} = 0.\dot{0}76923$	
	$\frac{2}{13} = 0.\dot{1}53846$
$\frac{3}{13} = 0.\dot{2}30769$	
$\frac{4}{13} = 0.\dot{3}07692$	
	$\frac{5}{13} = 0.\dot{3}84615$
	$\frac{6}{13} = 0.\dot{4}61538$
	$\frac{7}{13} = 0.\dot{5}38461$
	$\frac{8}{13} = 0.\dot{6}15384$
$\frac{9}{13} = 0.\dot{6}92307$	
$\frac{10}{13} = 0.\dot{7}69230$	
	$\frac{11}{13} = 0.\dot{8}46153$
$\frac{12}{13} = 0.\dot{9}23076$	

In the left column, the digits 076923 occur six times; likewise the digits 153846 on the right.

Naturally, initial digits for left and right side are in ascending order: 0,2,3,6,7,9, and 1,3,4,5,6,8.

Cyclic order is preserved in both groups:



The arrow-lines link the first & last fractions; second & second last; third & third last, in both brackets of digits.

The first three digits, 076, of $\frac{1}{13}$ become the last three of $\frac{12}{13}$, and so on with all the others, and each pair of decimals adds up to $0.9999\dots = 0.\dot{9} = 1$; also each pair of opposite single digits in the circles adds to 9.

FURTHER WONDERS

CALCULATORS lack display space to give all the digits for the repeating decimals of the reciprocals of prime numbers from 13 onwards, namely 17, 19, 23 etc, so, by long division, discover for yourself something unexpected about the number of repeating digits in them — something that did not apply to 13. (Both 13 and 7 have six repeating digits.)

If you can't work it out, ring the Help Line, 02 4829 0297, for the author. My phone is always on recorder, so start speaking and if I am here and you identify yourself, I shall answer at once; otherwise, leave your phone number.

Father James Tierney

The Finding in the Temple

OUR LADY lost her Holy Child;
It was three days all told.
She lost Our Lord when He was twelve,
Which is not very old.

2. Saint Joseph sought to find Our Lord,
And night and day he tried.
Saint Joseph did not sleep a wink,
Our Lady cried and cried.

3. But in the Temple finally
Saint Joseph went and found Him.
Our Lady and Saint Joseph ran
And threw their arms around Him.

4. Our Lady asked: "Son, why was this?"
He said: "Could you not guess?
Did you not know I was about
My Father's business?"

5. His parents found Our Lord at work
Instructing, if you please,
The learned Doctors of the Law,
The Scribes and Pharisees.

6. The Scribes and Pharisees were all
Astonished when they heard
Our Lord recite the prophecies
From Scripture, word for word.

7. It really was our Lord Himself
These prophets had foretold;
But He, of course, did not let on
When He was twelve years old.

8. He merely wanted at the time
To try the Doctors out
And give the Scribes and Pharisees
Some things to think about.

9. Our Lady was so glad and proud
To hear her little Son
Instructing such distinguished men
And charming every one,

10. Her heart was filled with joy again,
And so it came to be,
The Finding of Our Lord is called
A Joyful Mystery.

11. Our Lady and Saint Joseph, come,
Stay with me and mind me!
And if ever I get lost,
Hunt for me and find me! Fr Leonard Feeney SJ (see HO n. 73)

After Night Prayers

GOD KEEP the house from roof to floor,
The Twelve Apostles guard the door;
Four great angels round my bed,
Two my feet and two my head:
Matthew, Mark, Luke and John,
Bless this bed that I lie on.

Here I lie, for to sleep,
I give my soul to God to keep,
And if I die before I wake
I pray to God my soul to take. Monsunor Robert Hugh Benson

Hail, Mary (part 3)

WHEN OUR LORD was a little new baby
And lay on Our Lady's knees,
He heard the bees in the clover,
He heard the wind in the trees.

2. He remembered making the clover,
And telling the wind to blow,
He remembered putting the hum in the bee
And setting the trees to grow.

3. He remembered making Our Lady
To be Queen of Everything,
The Crown of the World, and His Mother,
He her son and her king.

4. The angels call her holy,
And we will do the same,
"Holy Mary, Mother of God,"
Our Lord made her name. Marigold Hunt

Mission Services

THOU didst leave Thy throne and Thy kingly crown
When Thou camest to earth for me,
But in Bethlehem's home there was found no room
For Thy holy nativity.

CHORUS: O come to my heart, Lord Jesus,
There is room in my heart for Thee.

2. Heaven's arches rang when the Angels sang,
Proclaiming Thy royal degree;
But in lowly birth didst Thou come to earth,
And in great humility.

3. The foxes found rest, and the birds had their nest
In the shade of the cedar tree;
But Thy couch was the sod, O Thou Son of God,
In the deserts of Galilee.

4. Thou camest, O Lord, with the living word
That should set Thy people free;
But with mocking scorn, and with crown of thorn
They bore thee to Calvary.

5. When heaven's arches ring, and her choirs shall sing
At Thy coming victory,
Let Thy voice call me home, saying, yet there is room,
There is room in my heart for Thee. Emily E.S. Elliott

Come, sing with Holy Gladness

COME, sing with holy gladness,
High Alleluias sing,
Uplift your loud Hosannas
To Jesus, Lord and King;
Sing boys in joyful chorus
Your hymn of praise to-day,
And, sing ye gentle maidens,
Your sweet responsive lay.

O boys, be strong in Jesus,
To toil for Him is gain,
And Jesus wrought with Joseph
With chisel, saw and plane;
O maidens, live for Jesus,
Who was a maiden's Son;
Be patient, pure and gentle,
And perfect grace begun. J.J. Daniell