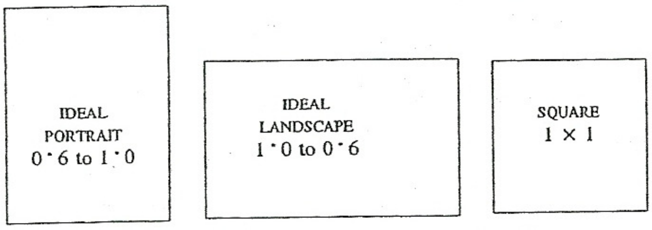
Fun with Figures

All you need is a pencil (with a rubber), paper and ruler

Fun with Shapes

Figure 1



PORTRAIT and Landscape are familiar terms when orienting A4 paper for photocopiers and computer printers. However, the above diagrams have the ratio 0.6 to 1.0, not the A4 size, 0 .7 to 1.0.

Picture frames when Landscape are in what ancient Greeks decided were ideal proportions for temples. It is called the Divine Proportion or Golden Number, roughly 3/5, or better, 5/8, (with reciprocals for Portrait).

Also, 3 to 5 were the proportions of the top and side views of the Ark of the Covenant, in Exodus 25:10.

Try measuring a few postage stamps and picture frames and check their proportions. Not all keep to the aesthetic best.

The Greeks calculated the Divine Proportion using geometry: draw a line in the sand, about a metre long and divide it at X with the square on the big part the same area as the rectangle made of the shorter part and the whole line. A rough result is when X divides the line into portions 0.6 and 0.4 of the whole line:-

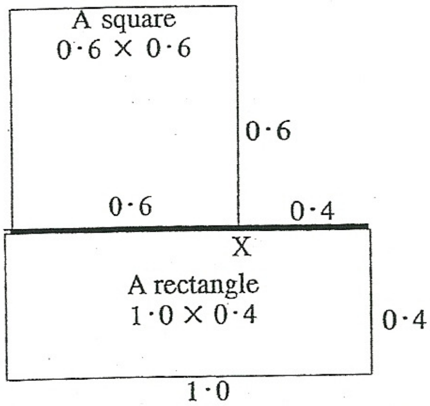
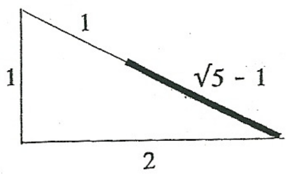


Figure 2

The area of the square is 0.36, which is roughly the same as the rectangle. It would be even better with parts of 0.625 & 0.375 in place of 0.6 and 0.4.

Advanced students can do this:-

Figure 3



Make 90° Δ with sides 2 and 1 units.
Hypotenuse is $\sqrt{5}$.
Subtract off 1 unit from it. Then the Divine Proportion, the ideal ratio, more accurate than 0·6, is half the rest of the hypotenuse, $\frac{\sqrt{5}-1}{2}$.

See overleaf for an algebraic method of calculation.

Fun with Numbers

Here you need Primary School maths, the part once called arithmetic. It uses easy algorithms: adding, dividing and turning fractions into decimals.

MAKE THE TABLE BELOW, THUS:-

Column A begins with 0 and 1. After that, generate new numbers by adding the two above them:-
0 + 1 = 1, 1 + 1 = 2, 1 + 2 = 3, 2 + 3 = 5,
3 + 5 = 8, 5 + 8 = 13, and so on.

Column B makes a fraction from pairs of numbers: take the number to its left in Column A and divide it into the one above it. Thus 5 divided into 3 is *Vs* and this is written alongside the 5.

Column C and Column D are decimals made from the fractions in Column B, taking it in turns like a steps-&-stairs. Here, decimals are to five places.

N.B. Students weak at maths should revise their multiplication tables by **doing the calculations by long division.** Calculators are best avoided in Primary School and should only be allowed for their speed and accuracy in the Secondary School by those capable of calculating without them.

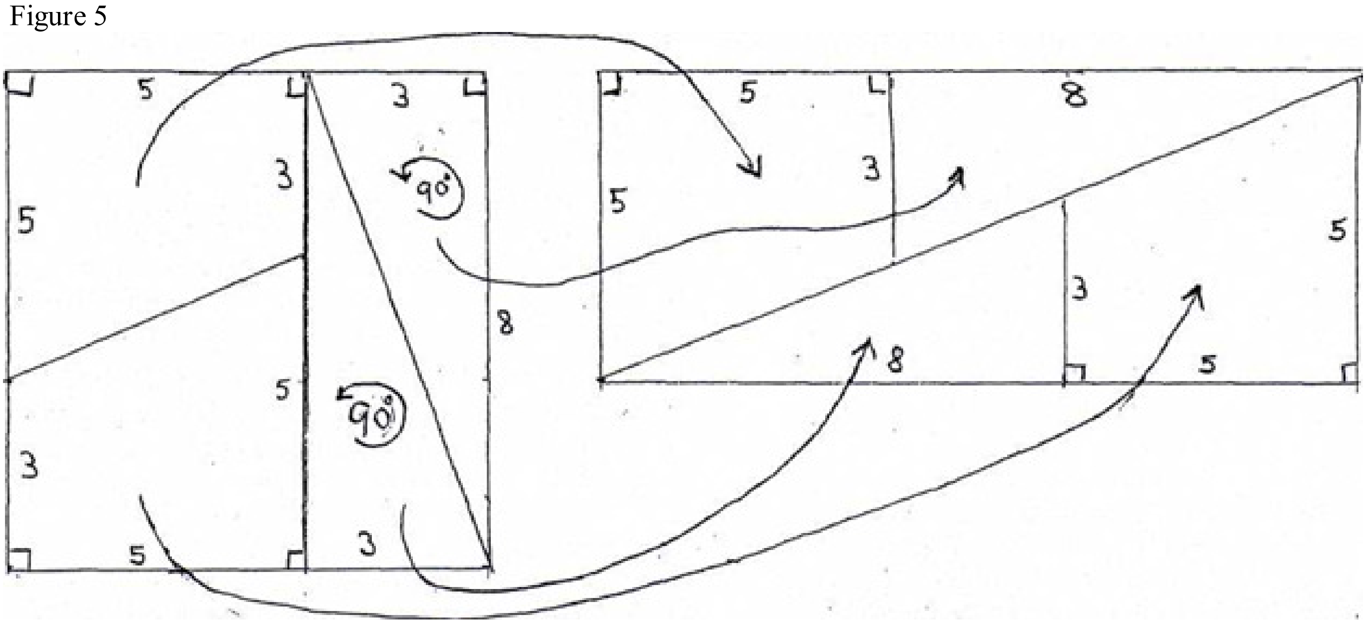
A	B	C	D
0			
1	*0/1*	0	
1	*1/1*		1.00000
2	*1/2*	0.50000	
3	*2/3*		0.66667
5	*3/5*	0.60000	
8	*5/8*		0.62500
13	*8/13*	0.61538	
21	*13/21*		0.61905
34	*21/34*	0.61765	
55	*34/55*		0.61818
89	*55/89*	0.61798	
144	*89/144*		0.61806
233	*14/233*	0.61803	
377	*233/377*		0.61804

What is the trend in Column C and Column D ? Comment on the final entries in these columns.

Fibonacci (c. 1170-1250) discovered the numbers in Column A and they are called the Fibonacci Series.

For *advanced students,* notice how both column are converging towards the Golden Ratio or Golden Number, the Divine Proportion, $\frac{\sqrt{5}-1}{2}$.

Figure 5



APPEARANCES ARE DECEPTIVE

Cut up a square 8 X 8 = 64 and re-arrange its parts as a rectangle 5 X 13 = 65. It seems that 64 = 65. So something is amiss! What is it? Work it out for yourself (or see explanation at the foot of the column).

This does not disparage the "hands-on" teaching of maths to the less enthusiastic students by cut-outs and models (see Handouts n. 18 on Pythagoras). Rather, it shows its limitations and invites students to rigorous proof, of which mathematics is the ultimate.

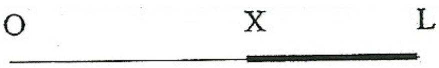
ALGEBRAIC calculation of Divine Proportion

As in Figure 2, draw a line OL, length one unit.

Mark a point X over half-way from the left end, call is distance x. Thus the rest of the line is 1 - x.

The square on x is to be equal to the rectangle whose sides are the whole line 1 and 1 - x.

Figure 6



We need to find x such that:

$x^2 = 1 \times (1 - x)$.

This is a quadratic equation $x^2 + x - 1 = 0$.

By the formula, $x = \dfrac{-1 \pm \sqrt{1^2 - 4 \times 1 \times (-1)}}{2 \times 1}$

$\therefore\ x = +\dfrac{\sqrt{5} - 1}{2}$ or $-\dfrac{\sqrt{5} + 1}{2}$.

Since $\sqrt{5} = 2{\cdot}23607$, the solutions are $x = 0{\cdot}61803$ or $-1{\cdot}61803$. (cf. bottom Figure 4)

The negative solution would put X left of O. So it is possible to divides the line OL externally and still fulfil the conditions.

The Deception in Figure 5

Calculate the "rise-over-run" of three nearly equal angles:-
for the hypotenuse of $\Delta = 3/8 = 0{\cdot}375$
for the real diagonal of rectangle $5/(8+5) = 5/13 = 0{\cdot}385$
for the sloping side of the trapezium $\frac{5-3}{5} = \frac{2}{5} = 0{\cdot}4$.
$\therefore$ the diagonal is just above the Δ and the sloping side of the trapezium, and likewise just below the upper trapezium and Δ, so there is a skinny cavity of area 1 square unit.
Note that 2, 3, 5, 8, 13 are Fibonacci Numbers (Figure 4) in which pairs, even second pairs, have nearly the same ratio.

REGULAR PENTAGONS

The Divine Proportion is the secret for constructing a regular pentagon — "regular" means all its sides are equal — and without using a protractor to get 108°.

To draw Figure 7, construct AABC and mark the point D on the hypotenuse (cf. Figure 3).

With centre A and radius AB (not AD) draw an arc to reach where E and F will be (F is on CA produced).

With D as centre and AB as radius, draw an arc to cut the first arc at E. The ADAE has angles of 72°, 72° and 36° (i.e. the base angles of this isosceles triangle are double the vertex angle. (A Euclidean proof for this will be posted on request.)

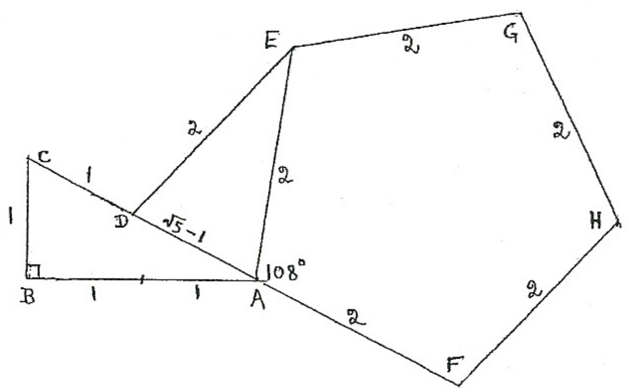
EA and AF are two sides of the required pentagon.

With A as centre and EF as radius, draw an arc sufficent to reach where G and H will be.

With centres E and F in turn, and radius EF, draw arcs to cut the other arc and thus fix points G and H.

Join up AEGHF to form the regular pentagon whose sides are equal to that of the original line AB.

Figure 7



NOTE FOR TUTORS at home or in school

This Handouts is meant for several lessons, not just one. As well as drawing the diagrams, it is best to cut out models in thick cardboard or thin hardboard.

Check the interesting history of Pythagoras and others for their anguish over square roots.