Cardinal Newman Catechist Consultants — 25th January, 2007 — *HANDOUTS* No. 38

"Clear, brief and easily assimilated by all"

Paper called A4

ACTIVITY

Even four year olds can join in the first part with homeschooled siblings.

TAKE two sheets of paper in the size called A4 — or give each pupil two sheets. Best use pages already printed on one or both sides, as your own recycling to save paper, trees and $.

Mark the top right corner of one sheet, A4.

Fold the other sheet. Press the crease flat. Use scissors, or slide a sharp knife, to cut along the crease. Rougher workers will rip the paper along the crease...

Mark one half sheet A5 in its top right corner.

Fold the other sheet of A5, press the crease flat, cut or slit or rip, and mark one half piece A6.

Continue to cut and mark till you get to A10.

Now stack the sheets, A4 at the bottom through to A10 on top, with all bottom left corners together.

OBSERVATION

WHAT do you notice about the top right corners? In teaching a class, re-phrase this question till most can grasp for themselves that the top right corners are all lined-up on a diagonal of the A4 sheet.

Also, mark the points on the A4 reached by A10, A9, through to A5, and draw the line linking them.

PUZZLEMENT

WHY do the corners of successive half-sheets line up on a straight line? If that does not strike you as surprising, it should.

Perhaps try another size in paper, and discover how special A4 is. For instance, cut off two sheets of A4, say at 250mm, to make a different shape.

Fold and cut one of them over and over, into smaller and smaller rectangles, and stack them on the other.

No longer do they all line up. The comers of the 3rd, 5th, 7th rectangles fall on the diagonal of the 1st; the corners of the 2nd, 4th, 6th on another line.

EXPLANATION

The way that A5 through to A10 fit on the diagonal of A4 suggests that these 7 pieces, though of different sizes, are all the same shape, because the sides are similarly proportioned.

NOT THE DIVINE PROPORTION

A4 etc lack the more aesthetically pleasing 5:8 Golden Ratio for portrait and landscape picture frames.

NUMBERING THE SIZES

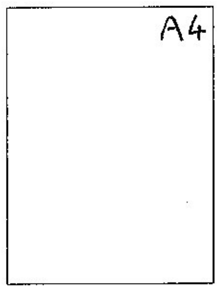
Why the numbers 4 to 10?

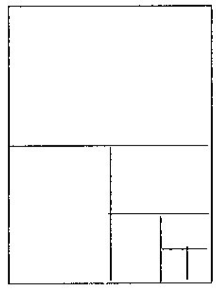
There is a much bigger size than A4 called A0. A0's first fold gives Al, and the second fold A2, hence down to the tenth fold, A10.

ECONOMY

Making the other sizes from a continuous halving of A0 means there is little waste. This saves trees, rubbish disposal, and money for both manufacturer and customers.

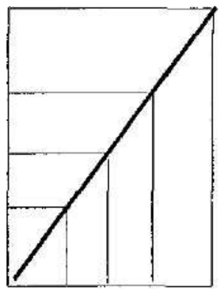
Further, the enlargement or reduction on a photo-copier from one size to another preserves the white space in the margins, and uses the paper efficiently.



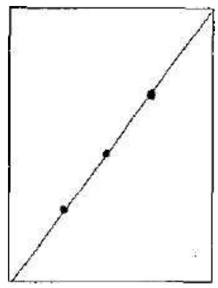


Younger children can pack the pieces together as a jigsaw puzzle

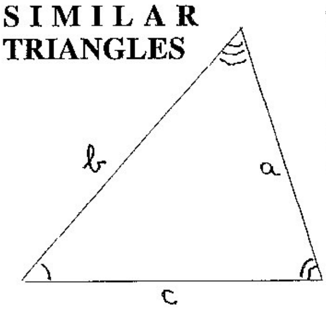
Lay a ruler on the top right corners (here we show only A4 down to A7)



Mark in the top right corners to check their line-up



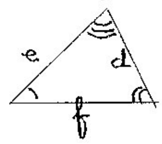
SIMILAR TRIANGLES



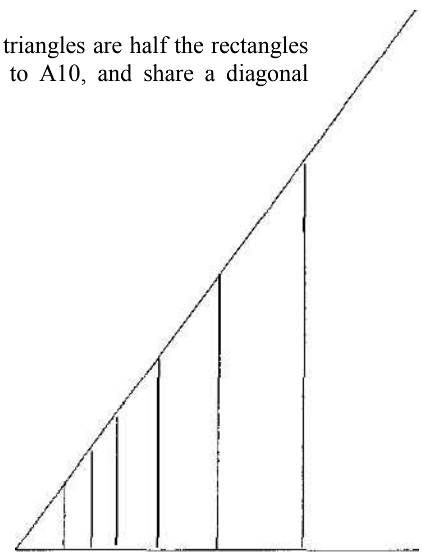
The angles of one triangle equal the angles of the other; the sides opposite them are proportional, but not equal:

$a : b : c = d : e : f$

$\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$



These triangles are half the rectangles of A4 to A10, and share a diagonal line



The heights of these triangles are the lengths of A4 - A10, and their bases are the widths.

Height-to-base proportions of these triangles are constant, just as the proportions of length to width of the A-sizes are constant

MEASUREMENTS

Check A4 and A5 sizes:

Paper	L=length	W=width
A4	297mm	210mm
A5	210mm	148mm

By long division or calculator:

Paper	L/W	W/L
A4	1.414	0.707
A5	1.414	0.707

(1) Area A4 = 2 x area A5, hence area A5 = $\frac{1}{2}$ area A4;

(2) the width W for A4 becomes the length L for A5;

(3) for both A4 and A5, the ratio L/W is the same, 1.414;

(4) also the length of A4 to length of A5 is 1.414; and check that the widths of A4 to A5, $\frac{210}{148}$, also equals 1.414;

(5) now prove that 1.414 = √2, and also that 0.707 = $\frac{1}{\sqrt{2}}$;

(6) the root of 2 is special: its reciprocal is half it!

Proof: $\frac{1}{\sqrt{2}} = \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{2}}{2}$ (rationalizing the denominator).

ALGEBRA IS PROOF...

Let x be that proportion of a long side L to a short side W which will be preserved on halving the paper:

A4 has x: 1 and A5 has 1 : $\frac{1}{2}$x because folding makes the short side into a long side. So x : 1 = 1 : $\frac{1}{2}$x

Cross multipy: $x^2 = 2$, so x = √2 = L/W.

So the secret of the paper's proportions is √ 2.

However, we prefer W/L to L/W because, later on, multiplying will be simpler than dividing in going from one entry to the next.

W/L= $\frac{1}{\sqrt{2}}$ =0.707

MAKE MORE MEASUREMENTS

Complete the tabulation and calculations of W/L.

Item	A4	A5	A6	A7	A8	A9	A10
Length	297						
Width	210						
W/L	0.707						

See the larger table for the correct answers for width and length. They are listed in most dictionaries, either under "paper", or in an appendix.

Is there a zig-zag pattern from one size to the next? Yes, a short side W becomes a long side L of the next,

What is notable about W/L for all sizes? Answer: The W/L ratios are all the same, within the order of accuracy of the measurements, which confirms the algebra above.

The bigger sizes in the A series, from A0 down to A4, follow exactly the same rules. Most pupils are familiar with A3, that it is double A4, and that to go from A4 to A3 on a photocopier needs an enlargement of 141 %, or a multiplication of 1.414, i.e. √2.

METRIC SIZED PAPER

Metric sizes mean not just measuring in metric units, but using whole numbers. For instance, an imperial measure for tea is one pound (1 lb), and measured metrically this is 454g. However, to be packaged metrically, the packed would have to be increased to 500g, a nice round number.

But 297 and 210 are not nice round numbers...

However, there is a second series of paper sizes called B0 to B10. Again, see the next table or the dictionary.

B0, the biggest sheet, is 1.414 metres x 1 metre. That is the closest such paper can get to being metric, since it cannot have both its sides in an exact metric measurement if the paper is to fold in half and preserve its proportions. L/W = *√2,* and each successive size in the B series is $^1/_{\sqrt{2}}$ times the previous one, in exactly the same fashion as in the A series. So although most of the numbers do not suggest it, the paper sizes are based on one metre. B-sizes begin with a roll one metre wide, which necessitates that the A-sizes begin from a roll 841mm wide.

RELATION OF A and B SIZES

From the A and B tables in a dictionary, a combined table can be constructed, thus:

Size	Length	Width	Size	Length	Width
B0	1414mm	1000*mm			
A0	1189	841			
B1	1000	707	B6	176	125*
A1	841	594	A6	148	105
B2	707	500*	B7	125	88
A2	594	420	A7	105	74
B3	500	353	B8	88	62*
A3	420	297	A8	74	52
B4	353	250*	B9	62	44
A4	297	210	A9	52	37
B5	250	176	B10	44	31*
A5	210	148	A10	37	26

The B-sizes fit neatly between the A-sizes, and this gives a much richer range of paper sizes.

RATIO OF SUCCESSIVE ITEMS ABOVE

Pupils can check the ratio of L to W for all entries above.

Next, examine the successive measurements in the columns of Length, and discover that it is the same ratio for the successive measurements in the Width column. For instance, look at the numbers marked with an asterisk* - each of those numbered with an * is half the previous one with a *, thus: 1000, 5000, 250, 125, 62 and 31.

Further, we already know that each entry in either the A or the B-sizes will be $\frac{1}{\sqrt{2}}$ times the previous one.

This would suggest that the successive entries for adjacent lines in the combined series above would be multiples of the reciprocal of the square root of 2, namely

$\frac{1}{\sqrt{\sqrt{2}}}$ = $1/\sqrt[4]{2}$ = $2^{-1/4}$ (2 raised to minus a quarter) = 0.841

Check the widths in the B-series: 1000 x 0.841 = 841mm.

Check that $0.841^2 = 0.707 = \frac{1}{2} \times 1.414 = \frac{1}{2}\sqrt{2} = \frac{1}{\sqrt{2}}$

MAKING THE TABLE OF A and B SIZES

Enter multiplicand 0.841 in the calculator; and press multiply (x).

Enter the Length of B0 as the multiplier, 141; press equals (=) and get 1189. Press (=) again, and get 1000; and so on to get the entire table of all the lengths of B0 and A0 down to B10 and A10. By pressing (=) repeatedly, we have chain multiplication, and the last answer becomes the next multiplier.

Now enter 1000, the Width of B0, as the multiplier; and generate the widths of all the items from B0 to A10.

To generate the B and A tables separately, use 0.707 as the multiplicand; the multiplier for the Lengths in B is 141, and for A 1189; the multipliers for their Widths 1000 and 841.

Round answers of (usually down) to the nearest millimeter. For greater accuracy, use 0.8408964 and 1414.2135.

CONCLUSION

For pupils who trigonometry, L/W = tan θ, where θ is the angle of the diagonal to the short side. See the similar but non-congruent triangles on the previous page.

Thus actarn $\frac{297}{210}$ also written as $\tan^{-1} \frac{297}{210}$ gives θ = 54°44'

Which is a constant for all A and B sizes of paper.

For the geometry to construct lines of length √2, $\frac{1}{\sqrt{2}}$, $2^{1/4}$ and $2^{-1/4}$

Contact Father Tierney (but not after 2030 hours when his phone is off the hook).

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